DISCONTINUOUS GALERKIN FEM FORMULATION FOR LINEAR THERMO-ELASTO-DYNAMIC PROBLEMS

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Abstract
The project's objective is to enhance the state of the art in the dynamic fracture modeling of thermo-elastic materials by studying the effects of temperature and rate dependence of the fracture properties on the resulting dynamic failure behavior. The project includes the development of (*a*) a discontinuous Galerkin space-time finite element method (DGFEM) for linear thermo-elasto-dynamic problems; (*b*) modeling the rate and temperature sensitive fracture properties via cohesive zone (CZ) models. The CZ modeling will include the study of fracture under two failure criteria, a critical crack opening displacement one and a maximum stress one. The project began December 1, 2004. Accomplishments to date are: (*i*) a DGFEM that is unconditionally stable; (*ii*) a computer code implementation of such FEM scheme capable of adaptive self-refinement; (*iii*) a new technique based on the immersed boundary method for the modeling of crack surfaces in FE calculations in which the crack representation is completely independent of the underlying FE grid. A paper reporting the formulation in question and companion calculations has been accepted pending reviews and three others are under development. The implementation of CZ models in FEM has yet to begin.

Objectives
This project intends to expand the understanding of the role of temperature in controlling the dynamic failure behavior of advanced materials subject to combined thermo-mechanical loading. This objective include both an improved the continuum-based modeling of the fracture properties of materials as well as the formulation and development of an unconditionally stable and adaptive FEM for the solution of the resulting governing equations.

Status of Effort and Accomplishments
The project began December 1, 2004 and the tasks accomplished thus far are as follows. The PI, along with two post-doctoral fellows (Dr. Dinara Khalmanova, from August 2004–July 2006 and Dr. Luca Heltai, from August 2006–present),

1. has formulated a space-time discontinuous Galerkin finite element (DGFEM) formulation for fully coupled linear thermo-elasto-dynamic problems that has been shown to be unconditionally stable; A C++ code implementing the DGFEM in question has been developed and is capable of adaptivity;

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14. ABSTRACT
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2. applied this formulation to the solution of various model problems in thermo-elasto-dynamics as well as thermo-elasto-dynamic phase transitions and fracture;

3. pioneered a new method for the representation of crack surfaces that (*i*) does not rely on interpolation function enrichment , (*ii*) allows the fracture surface to cut through the elements of the underlying finite element grid, and (*iii*) is relatively easy to implement alongside adaptivity;

4. begun the testing of the crack surface procedure just mentioned.

Some 2D and 3D Results. The convergence properties of the formulation were presented in a previous report and have been discussed in a paper which has been accepted for publication pending revisions. More recent results concern the application of the DGFEM developed by the PI to the determination of the energy release rate in a dynamic solid-solid phase transition process as well to some initial calculations concerning a stationary crack in a thermoelastic material. Concerning these results, Fig. 1 shows a bi-material bar consisting of a soft phase (left) which is made to grow at the expense of a stiffer phase (right) to simulate dynamic damage propagation. The bar is first stretched and then fixed at its ends.

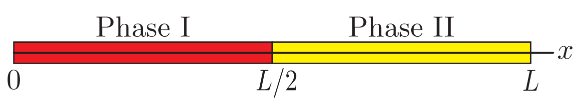

Figure 1: A one-dimensional bi-material.

Finally, the surface separating the two phases is made to advance according to a prescribed law. As soon as the interface moves, shock waves depart from the interface towards the bar end points and discontinuities in the stress and strain fields start propagating and causing the initially uniform temperature field to change. Figure 2(left) displays the strain-field

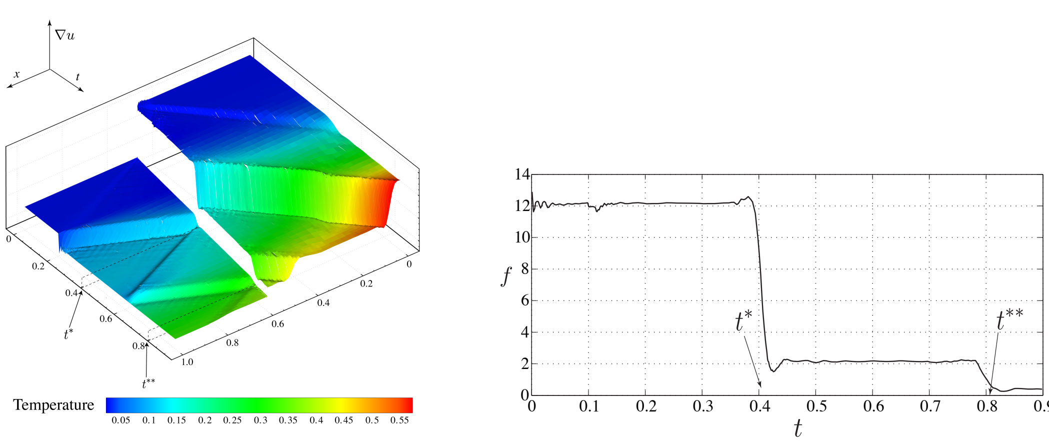

Figure 2: LEFT: 3D graph of strain field in the bi-material bar with moving phase transition interface. Color map corresponds to temperature field. The instants of time when the stress waves in the bar interact with the moving interface are denoted by t^* and t^{**}. RIGHT: Driving force on the moving interface in a bi-material bar as a function of time. It should be observed that the energy release rate suffers sudden drops at t^* and t^{**}.

solution in space-time as well as the corresponding temperature field (in color). In this problem, the energy dissipated at the interface was not used as a moving heat source and therefore thermo-elastic cooling (rather than heating) is observed at the moving interface. Figure 2(right) shows that the developed FEM is indeed effective in providing an accurate estimate of the forces driving the evolution of surfaces of discontinuity. Figure 3 displays the stress (left) and temperature (right) response of a thermo-elastic 2D bar which is fixed at one end, pulled at the other and then suddenly released. This simulation is again meant to illustrate the FEM developed can indeed deal well with highly time-discontinuous data. Some preliminary thermo-elastic fracture results are displayed in Fig. 4. As the crack tip is

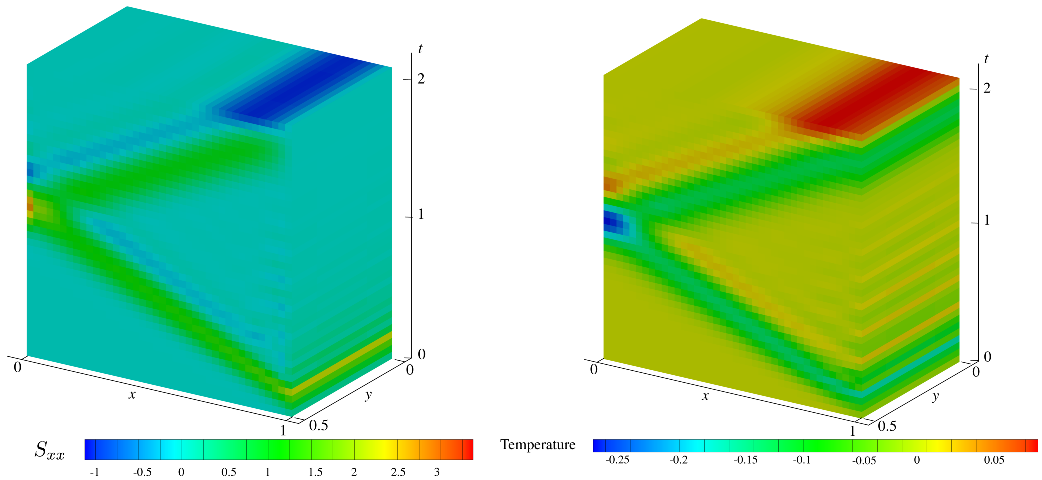

Figure 3: Space-time color graph of the normal stress S_{xx} (left) and temperature (right) in two-dimensional plate. In both cases the vertical axis is time. In this problem a 2D plate is fixed at one end and pulled at the other. The applied load at the free end is increased as a function of time and then suddenly released thus causing a shock wave to travel from the free end toward the fixed end of the bar (the green region tracks the motion of the shock wave and it reflection a the fixed end).

kept stationary in this simulation, thermo-elastic cooling at the crack tip is expected.

Recent Developments Recently, the project's efforts have been focused on the development of an efficient crack surface representation in FEM so as to have (*i*) complete independence between crack surface representation and the underlying finite element grid; and (*ii*) the implementation of the crack surface representation impact the adaptivity infracture of the finite element code as little as possible. Promising hp-methods are available based on the partition of unity that use interpolation enrichment as a way to allow for the representation of jump discontinuities within the elements in the underlying grid. These methods do require a certain amount of infrastructure to manage the interpolation and test function enrichment process. While interested in the implementation these techniques, the PI is also interested in developing a more efficient approach that does not require the enrichment overhead. In collaboration with his post-doc, Dr. Luca Heltai, the PI has been exploring a new crack surface representation strategy based on the finite element implementation of the

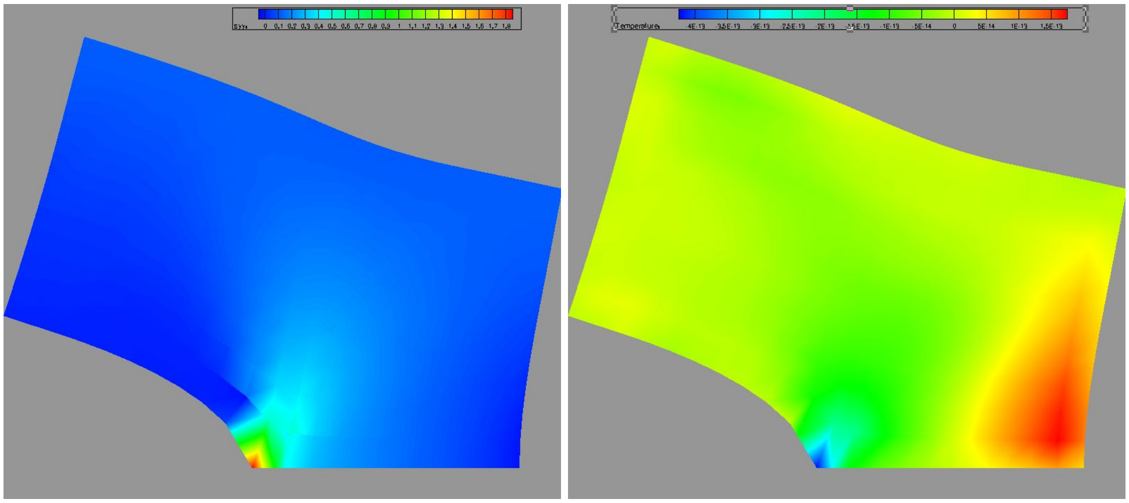

Figure 4: Color graph of stress component perpendicular to the crack plane (left) and of the temperature (right) in a rectangular plate (only a half of the plate is shown).

immersed boundary (IB) method (Boffi et al., 2007; Peskin, 2002). The basic idea of the proposed method is to model a crack surface as the singular support of a distributional force field causing a displacement discontinuity across the crack while preserving the continuity of the traction field. In this way, the representation of a crack does not require any cutting or other topological modification of the underlying finite element grid. The proposed IB based crack surface representation method has been applied to a simple two-dimensional static fracture problem in which a linear elastic isotropic panel with two cracks is loaded in tension. Referring to Fig. 5, the crack to the left has been modeled by a node release technique,

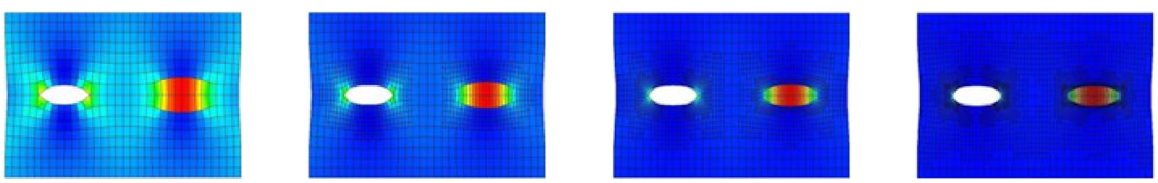

Figure 5: Two dimensional static fracture example. An isotropic linear elastic panel is subject to extension in the vertical direction. Two cracks are present: (left) one modeled via node release and (right) one represented as a Dirac delta distribution supported over a straight segment (equal to the crack segment on the left). The four images presented differ only by the level of adaptive refinement.

whereas the crack to the right has been modeled using the proposed IB technique. In both cases h-refinement was used. Notice how the case corresponding to the IB representation of the crack the mesh is not cut at all. The system simply deforms as if a crack were present, the crack presence being enforced via a Dirac delta distribution with support over the line along which the crack is defined.

Interactions

The following seminar and conference presentations acknowledging AFOSR support have been made:

1. F. Costanzo (2005), “A Discontinuous Galerkin Space-Time Formulation for Linear Elasto-Dynamics With Moving Surfaces of Discontinuities”, invited seminar, The University of California-Berkeley, Department of Mathematics.
2. D. K. Khalmanova. and F. Costanzo (2006), “Discontinuous Space-Time Galerkin Finite Element Method in Linear Dynamic Fully Coupled Thermoelasticity Problems with Strain and Heat Flux Discontinuities,” ECCM-2006, III European Conference on Computational Mechanics, Lisbon, Portugal, June 5–9.
3. D. K. Khalmanova and F. Costanzo (2006), “Discontinuous Space-Time Galerkin Finite Element Method in Linear Dynamic Fully Coupled Thermoelastic Problems with Strain and Heat Flux Discontinuities,” WCCM-VII, 7th World Congress on Computational Mechanics, Los Angeles, California, July 16–22.
4. F. Costanzo (2007), “A Discontinuous Galerkin Space-Time Formulation for Linear Elasto-Dynamics With Moving Surfaces of Discontinuities”, invited seminar, The Pennsylvania State University, Department of Mathematics.
5. L. Heltai (2007), “Distributional Body Force Densities in Finite Element Approximations of Continuum Mechanics Problems”, invited seminar, The Pennsylvania State University, Department of Mathematics.
6. L. Heltai (2007), “Distributional Body Force Densities in Finite Element Approximations of Continuum Mechanics Problems”, invited seminar, Department of Mathematics, University of Maryland.
7. L. Heltai and F. Costanzo (2007), “The use of Distributional body forces to enforce cracks in elastic materials”, to be presented at the *Minisymposium 105—Session 1: Numerical Techniques for the Modeling of Failure in Solids* within the 9th US National Congress of Computational Mechanics, San Francisco (CA), July 23–26.

Interaction with other AFOSR sponsored activities: This research program has benefitted and will continue to benefit from the interaction between the PI and Prof. Jay R. Walton (Department of Mathematics, Texas A&M University), also sponsored by AFOSR.

Interaction with other programs:

1. The PI is a Co-PI on a ONR-sponsored MURI program for the study of the failure behavior of rocket nozzles. This MURI program is strongly benefiting from the work reported herein. In turn the present AFOSR project has taken advantage of much of the code development done by the Ph.D. student supported by the MURI project.
2. The PI has begun collaborations with Prof. Jinchao Xu (Distinguished Professor of Mathematics, Penn State) and Prof. Ludmil Zikatanov (Associate Professor of Mathematics, Penn State), both experts in multi-grid methods, to further develop his FEM approach. Profs. Xu and Zikatanov have shown great interest in the intrinsic multiscale nature of the problem considered in the this grant and believe our collaboration is a great opportunity to demonstrate the benefits of multi-grid techniques. In order to

foster such a collaboration, a new seminar series was created within the Mathematics Department at Penn State with the specific objective to educate the students, post-docs, and faculty involved in the project.

3. The PI has also begun collaboration with Prof. Sefano Mariani of the Politenico di Milano (Milan, Italy). Prof. Mariani is the author of many publications dealing with the implementation of the extended FEM (XFEM) in fracture and damage mechanics. The objective of the collaboration is the implementation of XFEM methods in DGFEM code developed by the PI. Prof. Mariani has spent almost over two months visiting with the PI at Penn State working on the implementation of a fully coupled thermo-mechanical XFEM based code.

References

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Peskin, C. S., 2002. The immersed boundary method. In: Acta Numerica, 2002. Cambridge University Press.

Personnel Supported During the Duration of Grant

Dinara K. Khalmanova: Postdoc, Engineering Science and Mechanics Department, The Pennsylvania State University, University Park, PA 16802, August 2004–July 2006.
Luca Heltai: Postdoc, Engineering Science and Mechanics Department, The Pennsylvania State University, University Park, PA 16802, August 2006–August 2007.
Francesco Costanzo: PI, Engineering Science and Mechanics Department, The Pennsylvania State University, University Park, PA 16802.

Publications

KHALMANOVA, D. K. and F. COSTANZO (2006), “A Space-Time Discontinuous Galerkin Finite Element Method for Fully Coupled Linear Thermo-Elasto-Dynamic Problems with Strain and Heat Flux Discontinuities”, *Computer Methods in Applied Mechanics and Engineering*, accepted pending revisions.

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